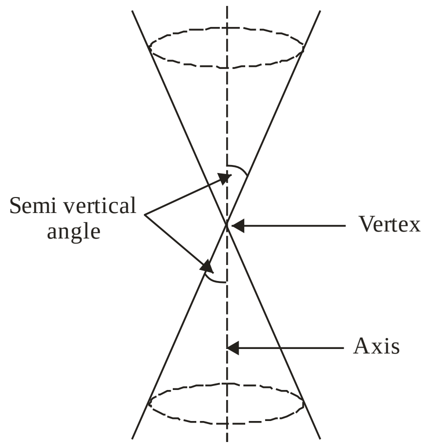
Conics

Introduction

We have studied Line and Circle in the earlier chapter and in the beginning of this chapter. We shall now study about three more curves viz. Parabola, Ellipse and Hyperbola which are called Conics or Conic Sections.



The figure shows a right circular cone with two parts meeting at the vertex and extending indefinitely both ways. Sections of such a cone by a plane in different positions with respect to its axis and the vertex give different types of curves. These curves are known as Conic Sections or Conics.

We shall now turn to the analytical aspect of these conics rather than bringing the cone and its intersection.
Let us define the conics.

Definition of Conic

A conic is the locus of a point such that the ratio of its distances from a fixed point and from a fixed straight line is a constant.
The fixed point (S) is called the focus, the fixed line (d) is called the directrix, the constant ratio (e) is called eccentricity and this property is called Focus-Directrix Property.
Since e is the ratio of distances, it is non-negative.
We get different conics according to different values of e.

If $e = 1$, the conic is called Parabola.
If $0<e<1$, the conic is called Ellipse.
If $e>1$, the conic is called Hyperbola.

Parabola

Definition:

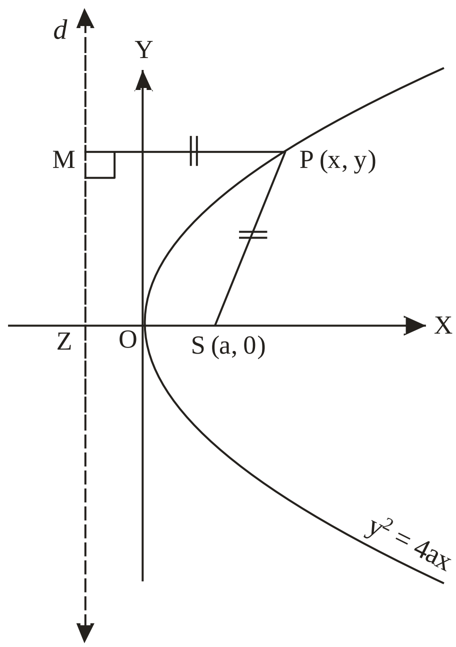
Parabola is the locus of a point which is equidistant from a fixed point S and a fixed line d. Thus parabola is a conic whose eccentricity is 1.

The fixed point S is called the focus and the fixed line d is called the directrix of the parabola. This definition is also called the focus-directrix property of the parabola.

Standard Equation of Parabola

Equation of the parabola in the standard form is $y^2 = 4ax$.

Sketch



Form of the Parabola $y^2=4ax$, $a>0$

i. The focus of the parabola is S(a, 0).

ii. Equation of the directrix is $x+a = 0$.

iii. Symmetry: If we fold the parabola along the X-axis, then the two parts of the parabola (one above the X-axis and the other below the X-axis) coincide. Hence the parabola is symmetrical about the X-axis.
i.e. the X-axis is the axis of the parabola.

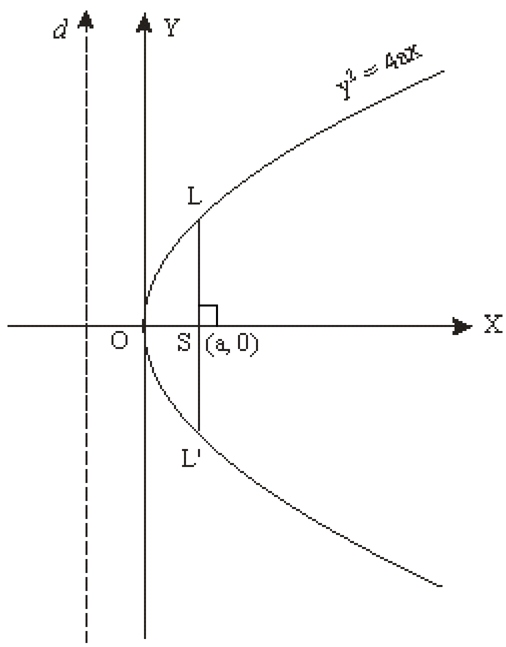
Here we observe that $y^2=4ax$. Hence $y = \pm\sqrt{4ax}$. This shows that for every positive value of x, there are two values of y which are negatives of each other. But for any value of y, there is one and only one value of x. Therefore the parabola is symmetrical about the X-axis but not about the Y-axis.

iv. Vertex: If a curve is symmetrical about a line, then this line is an axis of the curve. Further if this axis cuts the curve at a point, then this point is called a vertex of the curve.

The X-axis is the axis of the parabola and it cuts the parabola at the origin. Hence the origin is the vertex of the parabola.

v. Extent: y is real if $x \geq 0$. Hence the parabola lies wholly to the right of the Y-axis. Also as $x \to \infty$, $y \to \pm\infty$. Hence the parabola extends to infinity both ways on the right of the Y-axis.

vi. Latus Rectum: The chord of the parabola through its focus and perpendicular to its axis is called the latus rectum of the parabola.



Let LSL' be the latus rectum of the parabola $y^2=4ax$.

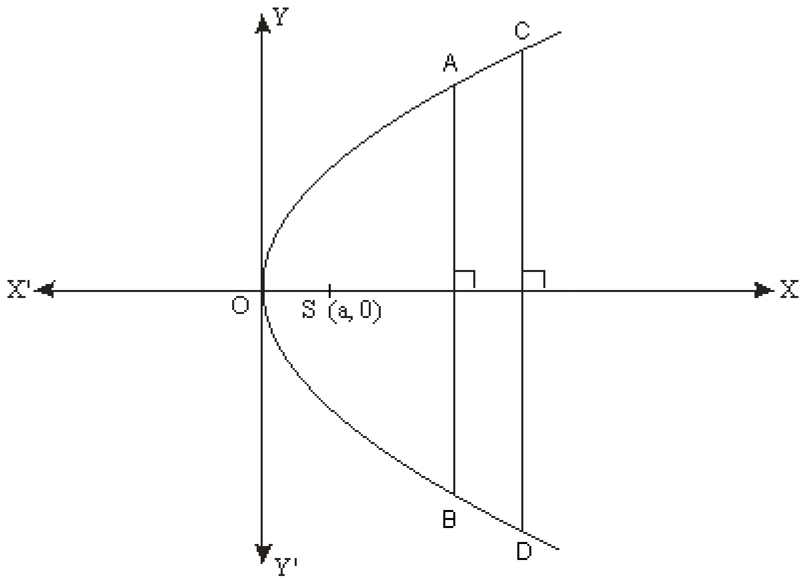
The end points (extremities) of the latus rectum are L (a, 2a) and L' (a, −2a)

The length of the latus rectum = 4a.

Note that this is the coefficient of x in the equation of the parabola.

vii. Double ordinate is a chord of the parabola perpendicular to the axis of the parabola. Its end points are of the form

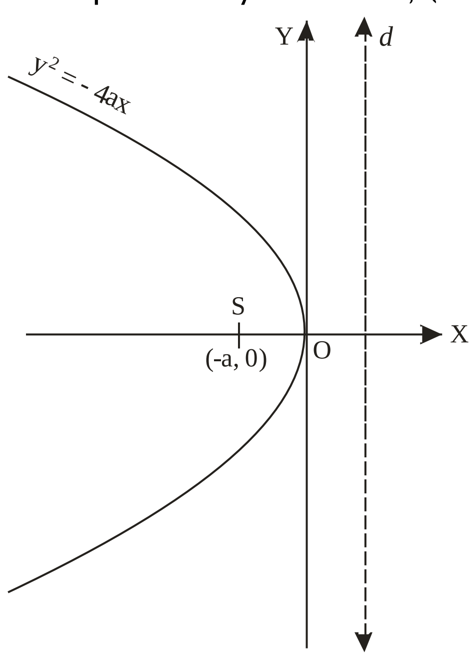
(x_1, y_1), $(x_1, -y_1)$.



AB, CD are double ordinates.

The other Parabolas

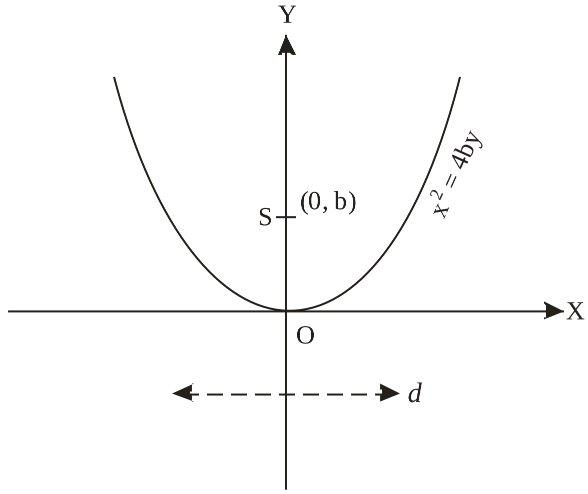
1. The parabola $y^2 = 4ax$; $a > 0$ is symmetrical about the X-axis, its vertex is at the origin and it opens out on the right of the Y-axis.
2. Now let us consider the parabola $y^2 = -4ax$; $(a > 0)$.



Its vertex is at the origin.

Its focus is S(–a, 0) and its directrix is the line x–a = 0. The X–axis is its axis and it opens out on the left of the Y–axis.

3. The parabola $x^2 = 4by$; $b > 0$ is shown in the following figure:

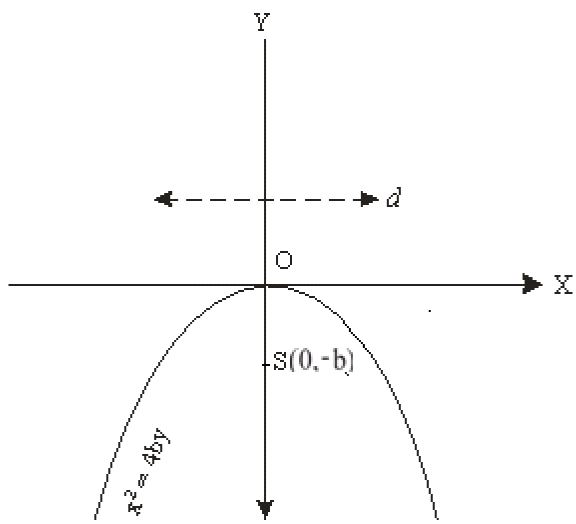


Its vertex is at the origin.

Its focus is S(0, b) and the directrix is the line y + b = 0.

This parabola is symmetrical about the Y–axis whose equation is x = 0. It opens out above the X–axis.

4. The parabola $x^2 = -4by$; $(b > 0)$ is shown in the following figure:



Its vertex is at the origin. Its focus is S(0, –b) and the directrix is the line y–b=0. This parabola is also symmetrical about the Y–axis and it opens out below the X–axis.

The above information about the four parabolas is given in the following table:

(Here $a>0$ and $b>0$).

Parabola	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4by$	$x^2 = -4by$
Focus	(a, 0)	(−a, 0)	(0, b)	(0, −b)
Directrix	$x + a = 0$	$x - a = 0$	$y + b = 0$	$y - b = 0$
Axis	X−axis	X−axis	Y−axis	Y−axis
Vertex	Origin	origin	Origin	origin
Latus Rectum	4a	4a	4b	4b
Ends of Latus Rectum	(a, 2a) (a, −2a)	(−a, 2a), (−a, −2a)	(2b, b), (−2b, b)	(2b, −b), (−2b, −b)
Opens out	to the right of the Y−axis	to the left of the Y−axis	Above the X−axis	Below the X−axis

Parametric Equations of Parabola:

The equation $y^2 = 4ax$ is called the Cartesian equation of the parabola.

The equations $x=at^2$, $y = 2at$ are called the parametric equations of the parabola.

If t is a parameter of a point P we denote it as P(t) and obviously its cartesian coordinates are $(at^2, 2at)$.

Focal Distance

The focal distance of a point P on the parabola is its distance from the focus S of the parabola. Seg SP is called the focal radius at P.

Let $P(x_1, y_1)$ be a point on the parabola $y^2 =4ax$.

Focal distance of $P(x_1, y_1)$ is $SP = PM = x_1+a$.

If P is $(at_1{}^2, 2at_1)$, then

Its focal distance $= x_1+a=at_1{}^2+a$.

If a chord of the parabola passes through its focus, then it is called a focal chord of the parabola.

Ellipse

Definition

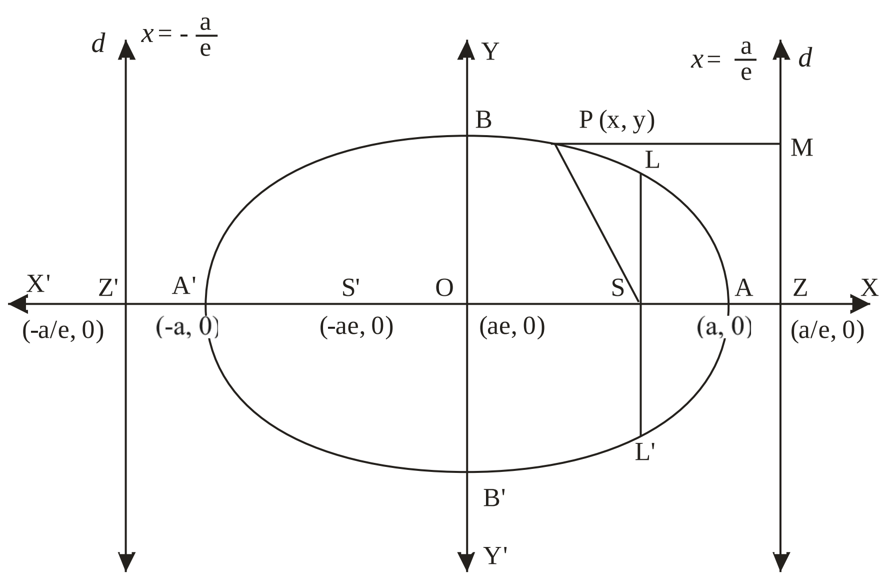
Ellipse is the locus of a point P whose distance from a fixed point S bears a constant ration e(<1) to its distance from a fixed line d. The fixed point S is called a focus, the fixed line d is called a directrix and the constant ratio e is called the eccentricity of the ellipse.

This definition is called focus-directrix property of the ellipse.

Standard Equation of Ellipse

The equation of an ellipse in the standard form is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$

Sketch



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Features of the Ellipse

- The Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ (a>b)$:

i. Symmetry:

(a) The ellipse is symmetrical about both the coordinate axes.

(b) Let $P(x_1, y_1)$ be a point on the ellipse.

Then $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1$

$\therefore \frac{(-x_1)^2}{a^2} + \frac{(-y_1)^2}{b^2} = 1.$

This shows that the point $Q(-x_1, -y_1)$ is also on the ellipse. In this case we say that the ellipse is symmetrical about the origin O (0, 0).

Also the midpoint of the chord PQ is (0, 0). In other words, every chord of the ellipse passing through the origin is bisected at the origin. In this case, we say that the origin is the centre of the ellipse. Hence ellipse is a central conic. Parabola has no centre, hence parabola is a non-central conic.

ii. Vertices and Axes:

A(a, 0); A'(-a, 0); B(0, b) and B'(0, -b) are the vertices of the ellipse.

A'A = 2a is called the major axis and

B'B = 2b is called the minor axis of the ellipse.

Also a is its semi-major axis and b is its semi-minor axis.

iii. Foci and Directories: S (ae, 0) is a focus of the ellipse and $x=a/e$ is the equation of its directrix. Since the ellipse is symmetrical about the Y-axis, it has another focus S' (-ae, 0) and another directrix corresponding to this focus whose equation is $x = -a/e$.

The distance between the foci is 2ae.

The distance between the directrices is 2a/e?

iv. Extent:

The ellipse is a closed curve lying between the lines $x=\pm a$ and $y=\pm b$.

v. Eccentricity : We have taken $b^2=a^2(1-e^2)=a^2- a^2e^2$

$\therefore a^2 e^2 = a^2 - b^2$

$\therefore e^2 = \frac{a^2 - b^2}{a^2}$

$\therefore e = \frac{\sqrt{a^2 - b^2}}{a}$

This gives the eccentricity of the ellipse.

vi. Latus Rectum: A chord of the ellipse through its focus and perpendicular to its major axis is a latus rectum of the ellipse.

Since the ellipse has two foci, it has two latera recta (plural of latus rectum).

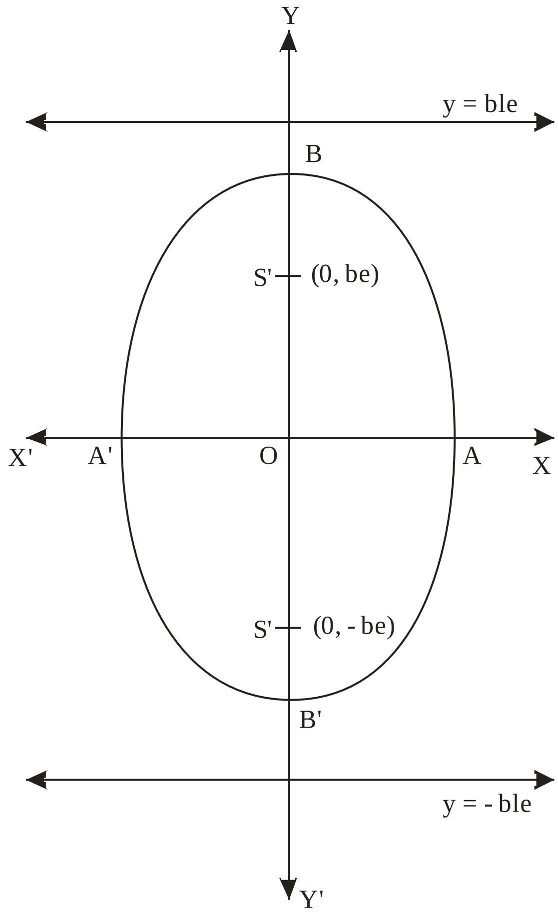
The extremities of the latus rectum are L $(ae, b^2/a)$ and L' $(ae, -b^2/a)$.

Length of the latus rectum = $2b^2/a$.

Similarly, the extremities of the latus rectum through the focus S'(−ae, 0) are (−ae, b^2/a) and

(−ae, −b^2/a).

- The Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $(a<b)$:



Now, let us consider the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ (a<b)$.

It is shown in the above figure.

This ellipse is also symmetrical about both the coordinates axes and about the origin.

(1) Its centre is (0, 0)

(2) $a^2 = b^2 (1-e^2)$

$\therefore e=\frac{\sqrt{b^2-a^2}}{b}$

(3) Its foci are (0, $\pm$be).

(4) Equations of the directrices are

$y=b/e$ and $y=-b/e$

(5) Its vertices are A(a, 0), A'(−a, 0),

B(0, b) and B'(0, −b).

(6) AA' = 2a is its minor axis

BB' = 2b is its major axis

a is the semi-minor axis and

b is the semi-major axis.

(7) Latus rectum = $2a^2/b$

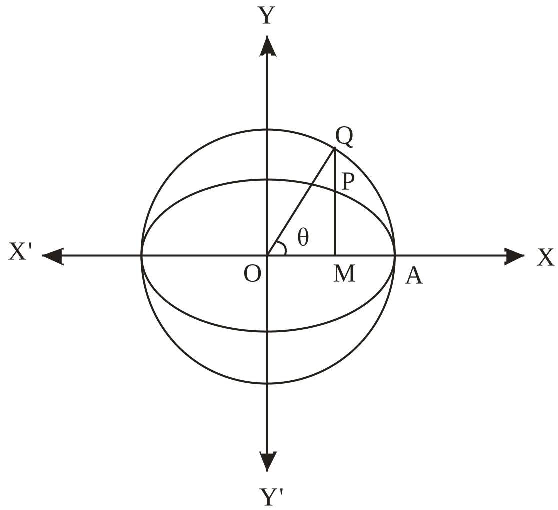
Extremities of the latera recta are

$(a^2/b, be)$; $(-a^2/b, be)$; $(-a^2/b, -be)$ and $(a^2/b, -be)$.

The above information about the two ellipses is given in the following table:

Ellipse	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $a > b$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $b > a$
Centre	(0, 0)	(0, 0)
Vertices	$(\pm a, 0), (0, \pm b)$	$(\pm a, 0), (0, \pm b)$
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Relation between a, b, e	$b^2=a^2(1-e^2)$	$a^2=b^2(1-e^2)$
Eccentricity	$e=\frac{\sqrt{a^2-b^2}}{a}$	$e=\frac{\sqrt{b^2-a^2}}{b}$
Equation of axes	Major axis: y = 0 Minor axis: x = 0	Major axis: x = 0 Minor axis: y = 0
Length of axes	Major axis: 2a Minor axis: 2b	Major axis: 2b Minor axis: 2a
Equation of directrices	$x=\pm\frac{a}{e}$	$y=\pm\frac{b}{e}$
Extremities of Latera Recta	$\left(ae, \pm\frac{b^2}{a}\right)$ $\left(-ae, \pm\frac{b^2}{a}\right)$	$\left(\pm\frac{a^2}{b}, be\right)$ $\left(\pm\frac{a^2}{b}, -be\right)$
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Distance between foci	2ae	2be
Distance between directrices	$\frac{2a}{e}$	$\frac{2b}{e}$

3) Parametric Equations of the Ellipse:



$(x^2/a^2)+(y^2/b^2)=1$ is the cartesian equation of an ellipse. On the major axis A'A as diameter, draw a circle. This circle is called the auxiliary circle of the ellipse. Its centre is the origin and radius is a. Hence its equation is $x^2+y^2=a^2$.

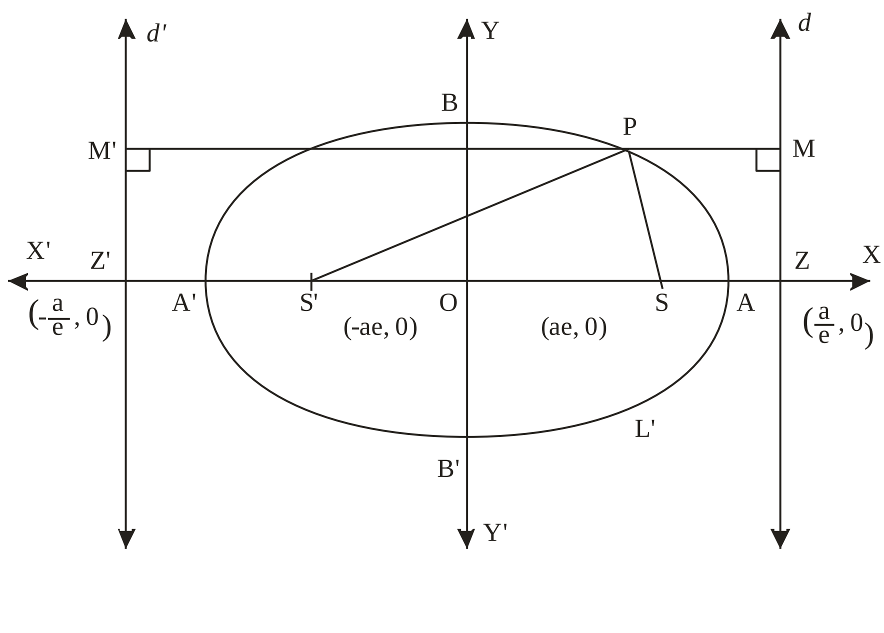
The parametric equations of the ellipse are

$x = a \cos \theta,\ y = b \sin \theta,\ (0^\circ \le \theta < 360^\circ)$

The angle θ is called the eccentric angle of the point P(x, y) on the ellipse.

Hence any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ can be taken as P (a cos θ, b sin θ).

4) Focal Distances:



Focal distance of a point on an ellipse is its distance from a focus. Since an ellipse has two foci S and S', the distances SP and S'P of a point P on the ellipse are the focal distances of P and segments SP and S'P are the focal radii at P.

Let the equation of an ellipse be $(x^2/a^2) + (y^2+b^2) = 1$ and $P(x_1, y_1)$ be a point on it. Then its foci are S(ae, 0) and S' (–ae, 0).

$$\therefore SP = \sqrt{(x_1 - ae)^2 + {y_1}^2}$$

$$= \sqrt{{x_1}^2 + {y_1}^2 - 2aex_1 + a^2e^2} \text{ and}$$

$$S'P = \sqrt{(x_1 + ae)^2 + {y_1}^2}$$

$$= \sqrt{{x_1}^2 + {y_1}^2 + 2aex_1 + a^2e^2}$$

These expressions for the focal distances SP and S'P are not simple. We derive convenient expressions for SP and S'P from the focus–directrix property of the ellipse as follows:

Draw segments PM and PM' perpendicular to the directrices d and d' respectively.

Then $PM = |x_1-(a/e)|$ and $PM'=|x_1 + (a/e)|$.

By the focus – directrix property of the ellipse,

$SP = e.PM = e|x_1 - (a/e)|=|ex_1 - a|$

Now $|x_1| < a$ and $e < 1$

$\therefore ex_1 < a \quad \therefore a - ex_1 > 0$

$\therefore SP =|ex_1 - a| = a - ex_1$

$S'P = e.PM' = e|x_1+(a/e)|$

$= |ex_1+a| = a+ex_1$.

Thus $SP=a-ex_1$ and $S'P=a+ex_1$. (Both are positive)

We observe that

(i) $SP + S'P = (a-ex_1) + (a+ex_1) = 2a$

(ii) Also $SP+S'P = e. PM + e. PM'$

$= e(PM+PM') = e. MM'$

Where MM' is the distance between the directrices

Which is 2a/e.

$\therefore SP+S'P = e(2a/e) = 2a$.

i.e. the sum of the focal distances of a point on the ellipse is equal to the length of the major axis.

(iii) If the parameter of P is θ_1, then $x_1=a\cos\theta_1$ and

$SP = a - ex_1 = a - e(a \cos \theta_1)$

$= a (1-e \cos \theta_1)$

and $S'P = a + ex_1 = a + e (a \cos \theta_1)$

$= a (1 + e \cos \theta_1)$.

Hyperbola

Definition

Hyperbola is the locus of a point P whose distance from a fixed point S bears a constant ration e (>1) to its distance from a fixed line d.

The fixed point S is called a focus, the fixed line d is called a directrix and the constant ratio e is called the eccentricity of the hyperbola.

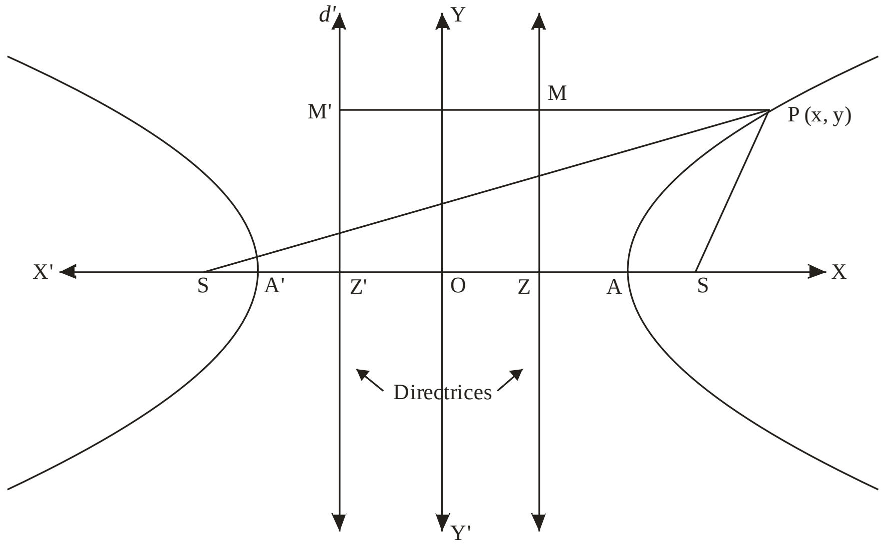
This definition is also called the focus-directrix property of the hyperbola.

Standard Equation of the Hyperbola

The equation of a hyperbola in the standard form is

$$(x^2/a^2) - (y^2/b^2) = 1.$$

Sketch



Features of the Hyperbola

[A] The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

i. **Symmetry**

(a)

The hyperbola is symmetrical about both the coordinate axes.

(b) Let $P(x_1, y_1)$ be a point on the hyperbola. Then

$$\frac{x_1^{\ 2}}{a^2} - \frac{y_1^{\ 2}}{b^2} = 1$$

$$\therefore \frac{(-x_1)^2}{a^2} - \frac{(-y_1)^2}{b^2} = 1$$

This shows that the point $Q(-x_1, -y_1)$ is also on the hyperbola. In this case we say that the hyperbola is symmetrical about the origin.

Also the midpoint of the chord PQ is O (0, 0). In other words, every chord of the hyperbola passing through the origin is bisected at the origin. In this case, we say that the origin is the centre of the hyperbola. Hence hyperbola is also a central conic.

ii. **Vertices and Axes**: The coordinate axes are the axes of symmetry of the hyperbola.

The Y-axis does not intersect the hyperbola.

A (a, 0) and A'(-a, 0) are the only two vertices of the hyperbola.

A'A = 2a is called the transverse axis of the hyperbola.

Now we take two points B(0, b) and B'(0, -b) on the Y-axis. Then B'B=2b is called the conjugate axis of the hyperbola.

iii. **Foci and Directrices**: S(ae, 0) is a focus of the hyperbola and x=a/e is the equation of its directrix. Since the hyperbola is symmetrical about the Y-axis, it has another focus S'(-ae, 0) and another directrix corresponding to this focus whose equation is x = -a/e.

The distance between the foci is 2ae.

The distance between the directrices is 2a/e.

iv. **Extent**: From the equation $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$,

We have, $\frac{x^2}{a^2}=1+\frac{y^2}{b^2}$ $\therefore \frac{x^2}{a^2}\geq 1$

$\therefore |x|\geq a$ $\therefore x \leq -a$ or $x \geq a$

This shows that no point of the hyperbola can lie in the region bounded by the lines $x = -a$ and $x = a$.

Also as $x \to \pm\infty$, $y \to \pm \infty$ and as $y \to \pm \infty$, $x \to \pm \infty$. Hence the hyperbola extends to infinity.

It is not a bounded curve.

v. **Eccentricity**: While deriving the standard equation of the hyperbola, we have taken

$b^2 = a^2 (e^2 - 1) = a^2 e^2 - a^2$

$\therefore a^2 e^2 = a^2 + b^2$

$$\therefore e^2 = \frac{a^2+b^2}{a^2}$$

$$\therefore e = \frac{\sqrt{a^2+b^2}}{a}$$

This gives the eccentricity of the hyperbola.

vi. **Latus Rectum**: The extremities of the latus rectum through focus S(ae, 0) are L(ae, b^2/a) and L'(ae, $-b^2/a$).

Length of the latus rectum = $2b^2/a$

Similarly, the extremities of the latus rectum through the focus S'(−ae, 0) are (−ae, b^2/a) and

(−ae, $-b^2/a$).

Rectangular Hyperbola

A hyperbola whose length of transverse axis is same as the length of conjugate axis is called a rectangular hyperbola.

Let the equation of rectangular hyperbola be

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad \text{............(1)}$$

The length of transverse axis = 2a

and length of conjugate axis = 2b

$\therefore 2a = 2b$, i.e., $a = b$

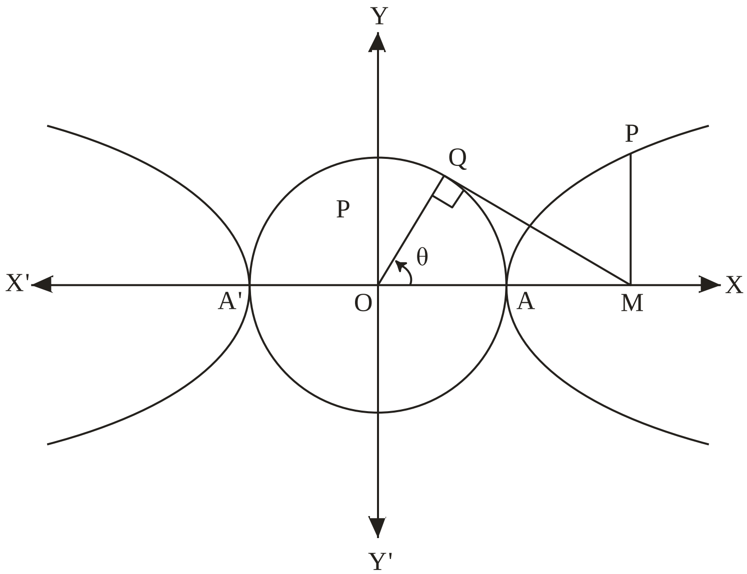
$\therefore$by (1), the equation of rectangular hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1, \quad \text{i.e. } x^2 - y^2 = a^2$$

$$\text{and its eccentricity} = e = \frac{\sqrt{a^2 + a^2}}{a} = \frac{a\sqrt{2}}{a} = \sqrt{2}$$

Parametric Equations of the Hyperbola:

$(x^2/a^2) - (y^2/b^2) = 1$ is the cartesian equation of the hyperbola.



On the transverse axis A'A as diameter, draw a circle.

This circle is called the auxiliary circle of the hyperbola.

Its radius = a.

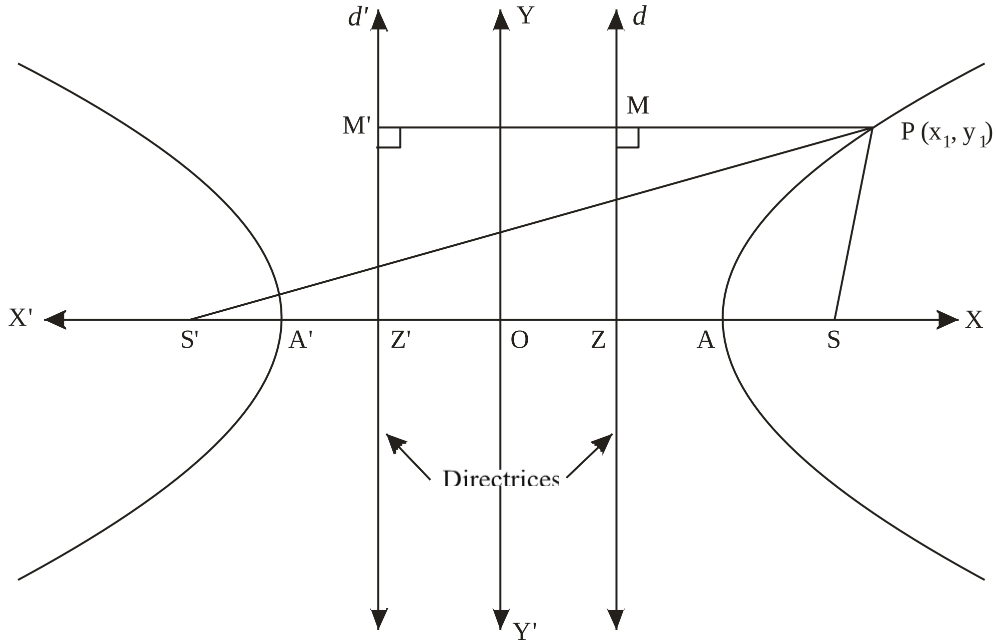
Let P(x, y) be a point on the hyperbola.

The parametric equations of hyperbola are $x = a \sec \theta$, $y = b \tan \theta$.

The angle θ is called the eccentric angle of the point P(x, y) on the hyperbola. Hence any point on hyperbola $(x^2/a^2) - (y^2/b^2) = 1$ can be taken as $P(a \sec \theta, b \tan \theta)$.

Focal Distances:

Focal distance of a point on a hyperbola is its distance from a focus. Since a hyperbola has two foci S and S', the distances SP and S'P of a point P on the hyperbola are the focal distances of P and segments SP and S'P are the focal radii at P.



Let the equation of a hyperbola be

$(x^2/a^2) - (y^2/b^2) = 1$ and $P(x_1, y_1)$ be a point on it. Then its foci are S(ae, 0) and S' (–ae, 0). The equations of its directrices d and d' are x = a/e and x = –a/e respectively.

Draw segments PM and PM' perpendicular to the directrices d and d' respectively.

Then $PM = |x_1-(a/e)|$ and $PM' = |x_1+(a/e)|$.

By the focus-directrix property of the hyperbola,

$SP = e.\ PM = e|x_1-(a/e)|$

$\quad = |ex_1-a|$

$S'P = e.\ PM' = e|x_1+(a/e)|=|ex_1+a|$

$\therefore |S'P - SP| = |ex_1+a-ex_1+a|$

$\quad = 2a$ = transverse axis.

i.e. the difference of the focal distances of a point on the hyperbola is equal to the length of the transverse axis.

If θ_1 is the parameter of P, then $x_1=a \sec \theta_1$ and

$SP = |e (a \sec \theta_1)-a|=a\ |e \sec \theta_1-1|$

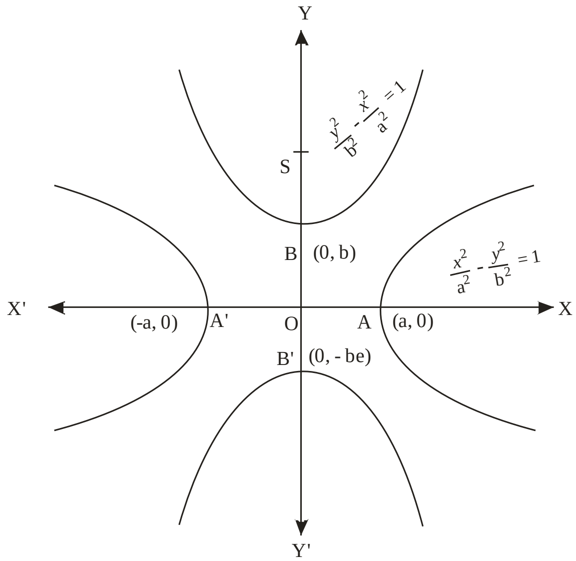
$S'P = |e (a \sec \theta_1) + a| = a\ |e \sec \theta_1+1|$.

B] Conjugate Hyperbola $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$:

Consider the hyperbola $(x^2/a^2) - (y^2/b^2) = 1$.

It has two branches and it does not intersect the Y-axis. Its vertices are A(a, 0) and A'(–a, 0). Its transverse axis = 2a.

Now consider the hyperbola $(y^2/b^2) - (x^2/a^2) = 1$



It has also two branches and its does not intersect the X-axis. Its vertices are B(0, b) and B'(0, –b), its transverse axis = 2b.

This hyperbola is called the conjugate of the hyperbola $(x^2/a^2) - (y^2/b^2) = 1$.

In fact these two hyperbolas are conjugates of each other.

If e' is the eccentricity of the hyperbola $(y^2/b^2) - (x^2/a^2) = 1$, then its foci are $(0, \pm be')$ and the equations of its directrices are $y = \pm b/e'$.

Its latus rectum = $2a^2/b$.

Also $a^2 = b^2(e'^2 - 1)$.

$$\therefore e'^2 = \frac{a^2 + b^2}{b^2}$$

$$\therefore e' = \frac{\sqrt{a^2 + b^2}}{b}$$

The above information about the hyperbola is given in the following table on next page:

Hyperbola	**$(x^2/a^2)-(y^2/b^2) = 1$**	**$(y^2/b^2) - (x^2/a^2) = 1$**
Centre	(0, 0)	(0, 0)
Vertices	$(\pm a, 0)$	$(0, \pm b)$
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Relation between a, b, e	$b^2=a^2(e^2-1)$	$a^2=b^2(e^2-1)$
Eccentricity (e)	$\frac{\sqrt{a^2+b^2}}{a}$	$\frac{\sqrt{a^2+b^2}}{b}$
Equation of axes	Trans. Axis: $y = 0$ Conj. Axis: $x = 0$	Trans. Axis: $x = 0$ Conj. Axis: $y = 0$
Length of axes	Trans. Axis: 2a Conj. Axis: 2b	Trans. Axis: 2b Conj. Axis: 2a
Equations of Directrices	$x = \pm a/e$	$y=\pm b/e$
Extremities of Latera Rectra	$(ae, \pm b^2/a)$ $(-ae, \pm b^2/a)$	$(\pm a^2/b, be)$ $(\pm a^2/b, -be)$
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Distance between Foci	2ae	2be
Distance between Directrices	$\frac{2a}{e}$	$\frac{2b}{e}$

MCQs

Q.1 ………… is an end point of latus rectum of parabola $5x^2=8y$

(a) $\left(\frac{4}{5}, \frac{-2}{5}\right)$ (b) $\left(\frac{-4}{5}, \frac{-2}{5}\right)$ (c) $\left(\frac{-4}{5}, \frac{2}{5}\right)$ (d) $\left(\frac{8}{5}, \frac{2}{5}\right)$

Q.2 Length of latus rectum of parabola $y^2=4ax$ through (2, −6) is………

(a) 9 (b) 18 (c) 6 (d) 12

Q.3 Parabola with focus (3, 0) and directrix $x+3=0$ is……

(a) $y^2=10x$ (b) $y^2=6x$ (c) $x^2=12y$ (d) $y^2=12x$

Q.4 Vertex of parabola is at origin. It passes through (3, −9). Y axis is its axis. Its equation is ………

(a) $x^2=y$ (b) $y^2 = x$ (c) $x^2 = -y$ (d) $y^2=-x$

Q.5 Point on $y^2=18x$ for which ordinate = 3 (abscissa) is ……

(a) (2, 6) (b) (6, 2) (c) (2, −6) (d) (2, 4)

Q.6 Parameter of a point on $y^2=8x$ is 3. The point is……

(a) (0, 0) (b) (18, −12) (c) (18, 12) (d) (8, 8)

Q.7 Point on $3y^2=16x$ with parameter $-\frac{1}{4}$ is

(a) $\left(\frac{4}{3}, \frac{-8}{3}\right)$ (b) $\left(\frac{1}{12}, \frac{-2}{3}\right)$ (c) $\left(\frac{1}{3}, \frac{4}{3}\right)$ (d) (0, 0)

Q.8 Parameter of point $\left(\frac{3}{2}, 6\right)$ on $y^2=24x$ is ……

(a) 2 (b) −1/2 (c) ½ (d) 1

Q.9 Focal distance of a point on $y^2=28x$ with parameter 14 is………

(a) 14 (b) 7 (c) 10 (d) 12

Q.10 Point on $y^2=4x$ with focal distance 5 is ……

(a) (−4, 4) (b) (4, −4) (c) (−4, −4) (d) (2, 4)

Q.11 vertex, focus and directrix of $y^2=4x+4y$ are

(a) (−1, 2) (0, 1), $x+2=0$ (b) (1, 2), (0, 2), $x+2=0$

(c) (−1, 2), (0, 2), $x+2=0$ (d) (−1, 2), (2, 0), $x+2=0$

Q.12 Vertex of parabola $x^2+4x+4y+16=0$ is……

(a) (2, 3) (b) (−2, 3) (c) (−2, −3) (d) (2, −3)

Q.13 Vertex, focus and directrix of parabola $4y^2+12x-12y+39=0$ are……

(a) $\left(\frac{-5}{2},\frac{3}{2}\right),\left(\frac{-13}{4},\frac{3}{2}\right), 4x-7=0$ (b) $\left(\frac{-5}{2},\frac{3}{2}\right),\left(\frac{-13}{4},\frac{-3}{2}\right), 4x-7=0$

(c) $\left(\frac{-5}{2},\frac{3}{2}\right),\left(\frac{-13}{4},\frac{-3}{2}\right), 4x+1=0$ (d) $\left(\frac{-5}{2},\frac{3}{2}\right),\left(\frac{-13}{4},\frac{3}{2}\right), 4x+7=0$

Q.14 Area of a triangle formed by lines joining the vertex of $x^2=12y$ to the ends of latus rectum is …… sq. units.

(a) 18 (b) 9 (c) 12 (d) 16

Q.15 A parabolic reflector is 20cm in diameter and 5cm deep, its focus is ……

(a) (4, 0) (b) (5, 0) (c) (10, 0) (d) (20, 0)

Q.16 The towers of a bridge, hung in the form of a parabola, have their tops 30 metres above the roadway and are 200 metres apart. If the cable is 5 metres above the roadway at the centre of the bridge the length of vertical supporting cable 30 metres from the centre is………

(a) 5.25m (b) 5m (c) 10m (d) 7.25m

Q.17 The eccentricity of an ellipse ……… is $\frac{\sqrt{7}}{4}$

(a) $9x^2+8y^2=72$ (b) $9x^2+16y^2=144$

(c) $4x^2+25y^2=100$ (d) $3x^2+4y^2=1$

Q.18 For ellipse $3x^2+4y^2=1$, length of latus rectum =

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{3}$ (c) 3 (d) $\frac{\sqrt{3}}{2}$

Q.19 For ellipse $4x^2+y^2=100$, length of major axis =

(a) 20 (b) 10 (c) 16 (d) 12

Q.20 Directrices of ellipse $9x^2+8y^2=72$ are ………

(a) $x=\pm 9$ (b) $y=\pm 9$ (c) $x=\pm\frac{1}{3}$ (d) $y=\pm\frac{1}{3}$

Q.21 The equation of an ellipse whose foci are at (±4, 0) and the eccentricity is $\frac{1}{3}$ is…….

(a) $\frac{x^2}{144}+\frac{y^2}{9}=1$ (b) $\frac{x^2}{144}+\frac{y^2}{128}=1$ (c) $\frac{x^2}{9}+\frac{y^2}{16}=1$ (d) $3x^2+y^2=1$

Q.22 Equation of ellipse in standard form whose major axis is 8 and eccentricity is 0.5 ….

(a) $3x^2+8y^2=48$ (b) $x^2+3y^2=16$ (c) $x^2+16y^2=1$ (d) $3x^2+4y^2=48$

Q.23 Ellipse with vertices (±5,0) and foci (± 4, 0) is…..

(a) $x^2+25y^2=25$ (b) $9x^2+25y^2=225$

(c) $2x^2+25y^2=50$ (d) $4x^2+25y^2=100$

Q.24 Point (6, 4) lies on ellipse whose eccentricity is $\frac{3}{4}$. Its equation is ....

(a) $x^2 + 2y^2 = 68$ (b) $x^2 + 4y^2 = 100$ (c) $3x^2 + 4y^2 = 96$ (d) $\frac{7x^2}{508} + \frac{4y^2}{127} = 1$

Q.25 Ellipse passing through the points (4, 3) and (6, 2) is ......

(a) $\frac{x^2}{11} + \frac{y^2}{2} = 1$ (b) $\frac{x^2}{64} + \frac{y^2}{9} = 1$ (c) $\frac{x^2}{52} + \frac{y^2}{13} = 1$ (d) $3x^2 + 4y^2 = 84$

Q.26 The eccentricity of the ellipse, whose distance between directrices is equal to 3 times the distance between foci, is .....

(a) $\sin 30^0$ (b) $\cos 30^0$ (c) $\tan 30^0$ (d) $\sin 90^0$

Q.27 Standard ellipse with $e = \frac{2}{3}$ and 5 as length of latus rectum is .....

(a) $\frac{4x^2}{81} + \frac{4y^2}{45} = 1$ (b) $\frac{4x^2}{81} + \frac{y^2}{45} = 1$ (c) $\frac{4x^2}{81} + \frac{4y^2}{15} = 1$ (d) $\frac{x^2}{81} + \frac{y^2}{45} = 1$

Q.28 The equation of the ellipse in standard form whose distance between foci is $4\sqrt{2}$ and the length of latus rectum is 4 is .....

(a) $x^2+2y^2 = 64$ (b) $x^2+2y^2 = 16$ (c) $x^2+4y^2 = 64$ (d) $2x^2+3y^2 = 36$

Q.29 For ellipse $(\pm 3, 0)$ are end-points of major axis and $(0, \pm 2)$ are end points of minor axis. Ellipse is .....

(a) $4x^2 + 9y^2 = 1$ (b) $4x^2 + 11y^2 = 44$ (c) $x^2 + 2y^2 = 10$ (d) $4x^2 + 9y^2 = 36$

Q.30 An ellipse has OB as a semi-minor axis, S and S' are its foci and $\angle SBS'$ is right angle, then find the eccentricity of the ellipse .....

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

Q.31 The focal distances of the point P $\left(5, 4\sqrt{3}\right)$ on $16x^2 + 25y^2 = 1600$ are....

(a) 7, 11 (b) 7, 13 (c) 8, 13 (d) 6, 11

Q.32 Point P on ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $(a\cos\theta, b\sin\theta)$. If S and S' are foci then SP. S'P =

(a) $a^2\cos^2\theta + b^2\sin^2\theta$ (b) $a^2\sin^2\theta + b^2\cos^2\theta$

(c) $a\sin^2\theta + b\cos^2\theta$ (d) $(a^2-b^2)\cos^2\theta$

Q.33 Point P $\left(\frac{\pi}{3}\right)$ on ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ is .....

(a) $\left(\frac{5}{2}, 2\sqrt{3}\right)$ (b) $\left(-\frac{5}{2}, 2\sqrt{3}\right)$ (c) $(1, 4)$ (d) $\left(\frac{5\sqrt{3}}{2}, 2\right)$

Q.34 Eccentric angle for point $\left(\frac{5}{2}, \frac{4\sqrt{3}}{2}\right)$ on ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ is

(a) 30^0 (b) 60^0 (c) 45^0 (d) 120^0

Q.35 For hyperbola $9y^2-4x^2 = 36$, e =

(a) $\sqrt{13/2}$ (b) $\sqrt{13}/2$ (c) $\sqrt{13}$ (d) 3

Q.36 For which hyperbola, length of latus rectum is $\frac{18}{5}$?

(a) $x^2 - y^2 = 4$ (b) $\frac{y^2}{4}-\frac{x^2}{9}=1$ (c) $\frac{x^2}{25}-\frac{y^2}{9}=1$ (d) $\frac{x^2}{36}-\frac{y^2}{64}=1$

Q.37 Hyperbola with focus (4, 0) and directrix x = 1 is .....

(a) $3x^2 - y^2 = 12$ (b) $4x^2 - 9y^2 = 36$ (c) $x^2 - 2y^2 = 1$ (d) $25x^2 - y^2 = 225$

Q.38 Hyperbola with $(\pm 7,0)$ as vertices and e = $\frac{4}{3}$ is .....

(a) $\frac{x^2}{49}-\frac{y^2}{343}=1$ (b) $\frac{x^2}{49}-\frac{9y^2}{343}=1$ (c) $\frac{x^2}{49}-y^2=1$ (d) $3x^2-4y^2=1$

Q.39 The equation of the hyperbola of given transverse axis whose vertex bisects the distance between the centre and the focus, is ......

(a) $x^2 - 3y^2 = a^2$ (b) $x^2 - 3y^2=3a^2$ (c) $3x^2 - y^2=3a^2$ (d) $x^2 - y^2=2$

Q.40 Hyperbola with $e=\sqrt{2}$ and for which distance between foci is 16, is .....

(a) $x^2 - y^2 = 16$ (b) $x^2 - y^2 = 64$ (c) $2x^2 - y^2 = 32$ (d) $x^2 - y^2 = 32$

Q.41 Hyperbola with vertices ($\pm$ 2,0) and foci ($\pm$ 3,0) is .....

(a) $5x^2 -14y^2 = 20$(b) $5x^2 - 4y^2 = 20$ (c) $x^2 - 2y^2 = 4$ (d) $5x^2 - 4y^2 = 20$

Q.42 Hyperbola in standard form passing through (3, 3) and with length of conjugate axis as 8 is .....

(a) $\frac{x^2}{144}-\frac{y^2}{16}=1$ (b) $\frac{x^2}{25}-\frac{y^2}{16}=1$ (c) $\frac{25x^2}{144}-\frac{y^2}{16}=1$ (d) $\frac{3x^2}{4}-\frac{y^2}{16}=1$

Q.43 Hyperbola with foci (0, $\pm$ 12) and length of latus rectum 36 is .....

(a) $x^2 - 3y^2 + 108 = 0$ (b) $3x^2 - y^2 = 108$
(c) $x^2 - 16y^2 = 64$ (d) $x^2 - 3y^2 = 108$

Q.44 The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ The equation of the hyperbola, if its eccentricity is $\sqrt{2}$ is.....

(a) $x^2 - y^2 = 7$ (b) $2x^2 - 2y^2 = 7$ (c) $3x^2 - 3y^2 = 4$ (d) $5x^2 - 5y^2 = 9$

Q.45 If the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide, then the value of b^2 =

(a) 14 (b) 12 (c) 81 (d) 7

Q.46 If e_1 and e_2 are the eccentricities of the hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$ then $\frac{1}{e_1^2}+\frac{1}{e_2^2}=1$

(a) 1 (b) 0 (c) 2 (d) –1

Q.47 Hyperbola is $x^2 - 4y^2 = 100$. If S, S' are foci and A is vertex of hyperbola, then SA. S'A =

(a) 100 (b) 5 (c) 10 (d) 25

Q.48 The x–coordinate of a point can be expressed as 3 times the sum of a non–zero number and its reciprocal and y–coordinate of the point can be written as 2 times the difference of that number and its reciprocal. The locus of all such points is hyperbola.....

(a) $2x^2 - y^2 = 72$ (b) $x^2 - 2y^2 = 72$ (c) $4x^2 - 9y^2 = 144$ (d) $x^2 - 3y^2 = 144$

Q.49 For ellipse focus is (1, 1) , $e = \frac{1}{2}$ and directrix is $x - y + 3 = 0$. Its equation is .....

(a) $7x^2 + 2xy + 7y^2 - 22x + 10y + 7 = 0$ (b) $7x^2 + 2xy + 7y^2 - 22x - 10y - 7 = 0$

(c) $7x^2 + 2xy + 7y^2 - 22x - y + 7 = 0$ (d) $7x^2 + 2xy + 7y^2 - 22x - 10y + 7 = 0$

Q.50 For ellipse $x^2 + 4y^2 + 2x + 16y + 13 = 0$, eccentricity and latus rectum are .....

(a) $\frac{\sqrt{3}}{2}, 1$ (b) $\sqrt{3}$, 2 (c) $\frac{\sqrt{3}}{2}, 4$ (d) $\frac{\sqrt{3}}{2}, 2$

Q.51 The eccentricity and equation of the ellipse with minor axis 2b, if the line segment joining the foci subtends an angle 2α at the upper vertex are .....

(a) $\cos\alpha$, $x^2 \cos^2\alpha + y^2 = b^2$ (b) $\sin\alpha$, $x^2 \sin^2\alpha + y^2 = b^2$

(c) $\sin\alpha$, $x^2 \cos^2\alpha + y^2 = b^2$ (d) $\sin\alpha$, $x^2 \cos\alpha + y^2 = b^2$

Q.52 The equation of the ellipse whose axes are parallel to the coordinate axes having its centre at the point (2,−3), one focus at (3, −3) and one vertex at (4,−3), is.....

(a) $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{3} = 1$ (b) $\frac{(x-2)^2}{3} + \frac{(y+3)^2}{4} = 1$

(c) $\frac{(x-2)^2}{49} + \frac{(y+3)^2}{16} = 1$ (d) $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{3} = 1$

Q.53 A man running round a racecourse notes that the sum of distances from the two flag-posts from him is always 26 metres and the distance between the flag-posts is 24 metres. The equation of the path traced out by the man is .....

(a) $\frac{x^2}{169} + \frac{y^2}{25} = 1$ (b) $\frac{x^2}{36} + \frac{y^2}{25} = 1$ (c) $9x^2 + 16y^2 = 144$ (d) $\frac{x^2}{26} + \frac{y^2}{24} = 1$

Q.54 The value of λ, if the equation $\frac{x^2}{2-\lambda} + \frac{y^2}{\lambda-5} + 1 = 0$ represents an ellipse =

(a) $\lambda < 5$ (b) $\lambda > 2$ (c) $2 < \lambda < 5$ (d) $\lambda \neq 2, \lambda \neq 5$

Q.55 The eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$ where B is an end point of a minor axis, S and S' are two foci such that ΔBSS' is an equilateral triangle is .....

(a) 0.2 (b) 0.5 (c) 0.75 (d) 0.8

Q.56 For standard hyperbola, length of conjugate axis = $\frac{3}{4}$ (length of transverse axis). Then e =

(a) 1.25 (b) 1.5 (c) 2.5 (d) 5

Q.57 Hyperbola with directrix 2x + y = 1, focus (1, 2) and $e = \sqrt{3}$ is .....

(a) $7x^2 + 12xy - 2y^2 - 2x - 14y - 22 = 0$ (b) $7x^2 + 12xy - 2y^2 - 2x + 14y - 22 = 0$

(c) $7x^2 + 12xy - 2y^2 + 2x + 14y - 22 = 0$ (d) $7x^2 + 12xy - 2y^2 + 2x - 14y + 11 = 0$

Q.58 Centre of hyperbola $9x^2 - 16y^2 - 18x + 32y - 151 = 0$ is .....

(a) (1, –1) (b) (1, 1) (c) (2, 1) (d) (0, 2)

Q.59 When λ varies, the point of the intersection of the lines $x\sqrt{3} - y - 4\lambda\sqrt{3} = 0$ and $\sqrt{3}\,\lambda x + \lambda y - 4\sqrt{3} = 0$ describes a hyperbola. Its e =

(a) $\sqrt{2}$ (b) 3 (c) $\sqrt{3}$ (d) 2

Q.60 If the eccentricity of the hyperbola $x^2 - y^2\sec^2\alpha = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2\sec^2\alpha + y^2 = 25$, then the value of α =

(a) π (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

Answer Key

1–c	11–c	21–b	31–b	41–b	51–c
2–b	12–c	22–d	32–b	42–c	52–d
3–d	13–d	23–b	33–a	43–a	53–a
4–c	14–a	24–d	34–b	44–b	54–c
5–a	15–b	25–c	35–b	45–d	55–b
6–c	16–d	26–c	36–c	46–a	56–a
7–b	17–b	27–a	37–a	47–d	57–b
8–c	18–d	28–b	38–b	48–c	58–b
9–a	19–a	29–d	39–c	49–d	59–d
10–b	20–b	30–c	40–d	50–a	60–b